

Performance Evaluation of a GI/M (a, b)/1 Queueing Model for a Diagnostic Laboratory with Multiple Working Vacations and Equipment Breakdown

E. Praveena¹, Dr. K. Julia Rose Mary²

¹Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India.

²Associate Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India.

¹praveenamaths1027@gmail.com

²juliakulandaisamy@gmail.com

ABSTRACT

This paper analysis a Non-Markovian queueing model applied to a diagnostic laboratory system, where patient samples arrive according to a general distribution single server pattern with batch service where server undergoes multiple working vacations with server breakdown. The laboratory begins processing only when at least “a” number of samples are available and can handle up to a maximum of “b” samples per batch. The testing equipment described as a server may enter multiple working vacation periods. Patient samples arriving follows General distribution and the service follows Exponential distribution. The breakdown that may happens during working vacation periods or during regular busy periods are considered in this model. The system is formulated through a discrete-time Markov chain at pre-arrival epochs. The steady state equation, steady state solution for the described model are derived to determine key performance measures known as mean queue length. Finally, this paper computes the numerical analysis of mean queue length with the types of breakdown to study how breakdown parameters influence laboratory congestion and service reliability. The results offer practical insights for diagnostic centres to optimize maintenance scheduling, batch processing, and equipment validity.

KEYWORDS: Batch service, Breakdown, General Arrival, Multiple working vacation, Regular busy period, Queue length, Reliability.

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INTRODUCTION

In modern healthcare environments, particularly in diagnostic and medical laboratory systems, managing the efficient flow of patient samples has become a critical operational challenge. Laboratories often receive unpredictable patient inflows from various sources such as outpatient departments, emergency units, or external clinics. Delays in sample processing can lead to diagnostic backlogs, delayed treatment decisions, and decreased patient satisfaction. Therefore, mathematical models that can effectively capture the dynamic behaviour of such systems are essential for improving service efficiency and reliability. Queueing theory provides a powerful analytical tool to study and optimize healthcare service processes involving randomness in patient arrivals, testing times, and equipment reliability. Diagnostic centres typically process biological samples in batches. For example, blood or biochemical samples are often analysed in groups using automated analysers to ensure cost-effective operation and reduce chemical usage. Such a system can be effectively modelled as a batch service queue, represented as GI/M(a,b)/1, where arrivals follow a general independent distribution (GI), and the service times are exponentially distributed (M). The parameters “a” and “b” represent the minimum and maximum batch sizes, respectively, that the analyser can process at one time. When fewer than “a” number of samples are waiting, the equipment remains idle or enters a state of reduced service capacity known as working vacation.

In a practical laboratory setting, testing machines do not always function continuously at full speed. Periods of lower workload, staff shortages, or scheduled maintenance often lead the equipment to operate at a reduced service rate, while still processing some samples. This scenario corresponds to queueing theory is known as a working vacation. During such periods, the analyser is not completely idle but continues to serve samples more slowly than usual. When multiple such reduced-capacity periods occur consecutively, the system is said to experience multiple working vacations (MWV). This concept accurately reflects real-life diagnostic environments where analysers periodically enter maintenance before resuming normal operation.

In addition to working vacations, breakdowns represent another major source of service disruption in healthcare diagnostics. Equipment failures can arise due to overuse, reagent shortages, software errors, or calibration faults. These breakdowns may occur during either regular busy periods or working vacation periods. Both types of breakdowns reduce the effective testing rate, increase the sample queue, and impact patient waiting times. Proper modelling of these interruptions is necessary for laboratory administrators to design preventive maintenance strategies and minimize overall service downtime.

To represent these realistic laboratory processes, this study proposes and analyses a GI/M(a,b)/1 queueing model with multiple working vacations and two types of breakdowns β_v and β_R . The model captures both the batch nature of diagnostic testing and the stochastic nature of interruptions. A discrete-time Markov chain approach is used at the pre-arrival epoch to derive steady-

state probabilities for the system's various operational states known as regular busy, working vacation, idle, and breakdown. Using these probabilities, key performance measures such as mean queue length (L_q), server utilization, and system reliability are obtained.

Furthermore, a numerical sensitivity analysis is carried out to examine how varying breakdown parameters β_v and β_R influence system behavior. The proposed model thus serves as a realistic analytical framework for healthcare managers and diagnostic centre operators. It enables quantitative evaluation of operational strategies such as adjusting batch sizes, optimizing maintenance frequency to minimize waiting times and improve testing efficiency. The study can further be extended to multi-server laboratory environments to enhance the quality of laboratory.

REVIEW OF LITERATURE

Queueing theory has been widely applied in modelling and analyzing systems where resources are shared among competing users such as computer networks, manufacturing systems, and service facilities. Telecommunication systems with packet transmission, Manufacturing systems with Batch Processing and Maintenance, Health care systems with Multiple Patient services are some of the real-life examples of the described model. Several studies have analyzed GI/M(a,b)/1 queues with bulk services.

The concepts of batch queue were first introduced through the research of Bailey (1954) [1]. The general bulk service rule was pioneered by Neuts (1967) [2] with Poisson arrival and general service time distribution. Easton G.D and Chaudhry M L (1982) [3] discussed about $E_k/M(a,b)/1$ queueing system and its numerical analysis. Chaudhry M. I and Templeton J.G.C (1983) [4] discussed A first course in bulk queueing. Choi and Han (1994) [5] analyzed the GI/ M (a,b) / 1 queue with multiple vacations. Servi. L.D and Finn S.G (2002) [6] have assessed M/M/1 queues with working vacations (M/M/1/WV). Non-Markovian Queueing Systems was discussed in Stochastic Models in Queueing Theory by Medhi (2003) [7].

William J. Gray (2004) [8] developed Queueing Model with Multiple Types of Server Breakdown. GI/M/1 working vacation model was examined by Baba (2005) [9]. Tian, Li and Zhang (2007) [10] provided a survey results of working vacation queues. Gross, D. and Harris, C.M (2008) [11] have proposed fundamentals of queueing theory. Zhang and Xu (2008) [12] have examined the M/M/1 queue with multiple working vacation. Julia Rose Mary and Afthab Begum (2009) [13] have analyzed the Markovian M/M(a,b)/1 queueing model under multiple working vacation and derived the steady state probability distribution and mean queue length for the model. Ghimere R.P and Ghimire S (2011) [14] studied heterogeneous arrival and departure M/M/1 queue with vacation and service breakdown.

Sushil Ghimire, R.P. Ghimire, Gyan Bahadur Thapa (2015) [15] examined the Mathematical models of the $M^bM/1$ bulk arrival queueing system. Steady state analysis of M/M (a,b)/MWV/BR queueing model was studied by Rajalakshmi, Pavithra, Julia Rose Mary (2016) [16]. Gopinath Panda, Abhijit Datta Banik and Dibyajyoti Guha (2018) [17] assessed Stationary Analysis and optimal control under multiple working vacation policy in a GI/M(a,b)/1 queue. Extended analysis and computationally efficient results for the GI/ M(a,b)/1 queueing system was analysed by Samanta S.K Bank.B (2020) [18]. Praveen Kumar Agarwal, Anamika Jain, Madhu Jain (2021) [19] have proposed M/M/1 Queueing Model with Working Vacation and Two Types of Server Breakdown.

Mohan Chaudhry and jing Gai (2022) [20] proposed analytic and computational analysis of GI/M(a,b)/c queueing system. A.N. Dudin, S.R. Chakravarthy, S.A. Dudin, O.S. Dudina (2023) [21] studied Queueing System with Server Breakdowns and Individual Customer Abandonment. Sundrapandian. S and Nandhini.S (2024) [22] developed the sensitivity analysis of a non-Markovian feedback retrial queue, reneging, delayed repair with working vacation subject to server breakdown. Lydia P and Julia. K (2024) [23] analyzed the performance of M/M(a,b)/1/MWV Queueing Model with the Busy Period Breakdown. Yi Zheng, Juxihong Julaiti, Guodong Pang (2024) [24] documented **Adaptive Service Rate Control of an M/M/1 Queue with Server Breakdowns**. This study extends the existing model of general arrival with batch service by developing the steady state analysis of GI/M (a,b)/ 1 queue with multiple working vacation and types of break down.

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METHODOLOGY

In this model the in the diagnostic laboratory process the analyser acts as a server that processes the patients samples in batches according to the general bulk service rule (GBSR) introduced by Neuts (1967). This rule states that analyser starts service only when there is at least "a" number of samples are in the system and maximum service capacity is "b" samples in a single batch. Regular service follows exponential distribution and it is denoted by the parameter μ_{RB} . After completing each service if the server finds at most "b" number of samples in the system then the server takes all "b" samples in a single batch to serve. If the analyser finds more than "b" then it takes first "b" number of samples as first batch for service. Thus, the service time of batches of size "k" ($a \leq k \leq b$) is assumed to be independent identically distributed. If there are less than "a" number of samples in the system then the server takes vacation. During vacation if the server finds the laboratory with more than "a" number of samples then the analyser starts the service with lower service rate μ'_{WV} which follows exponential distribution. The analyser repeats another working vacation if it finds less than "a" number of samples in the laboratory (i.e) multiple working vacation which is exponentially distributed and is assumed to be η . In this model the arrival is assumed to follow General distribution with the parameter λ . The model is considered for the server breaking down during working vacation and regular busy period which follow

Poisson distribution with a parameter β_v and β_R respectively. With the help of the above parameters, the model is analysed under Discrete-Time Markov Chain pre-arrival epoch and there by Mean queue length is also derived.

Let $t_n = 1, 2, 3, 4, \dots$ be the arrival time of the n^{th} number of samples. If arrival stream is empty then $t_0 = 0$. The inter arrival times $\{t_n, n = 1, 2, 3, 4, \dots\}$ are in-dependent and identically distributed with a general distribution function $G(t)$ with mean $1/\lambda$ and the Laplace Stieltjes Transform of $G(t)$ is denoted by $G^*(\theta)$ where $\theta \geq 0$. Let $N_q(t)$, be the number of patients samples waiting for testing in the queue at time t and $J_n(t) = 0, 1, 2$ where server is in idle vacation, working vacation and regular busy period respectively. In this model the process $\{(N_q(t_n - 0), J_n); n \geq 1\}$ is an embedded Markov chain with state space $C = \{(n \geq 0); J = 1, 2\} \cup \{0 \leq n \leq a-1; J = 0\}$.

The steady state queue size probabilities at a time t are defined by

$$\begin{aligned} P_{nR}(t) &= \lim_{k \rightarrow \infty} Pr\{N_q(t_k - 0) = n, J_n = 1\}, n \geq 0 \\ Q_{nW}(t) &= \lim_{k \rightarrow \infty} Pr\{N_q(t_k - 0) = n, J_n = 2\}, n \geq 0 \\ R_{nI}(t) &= \lim_{k \rightarrow \infty} Pr\{N_q(t_k - 0) = n, J_n = 0\}, 0 \leq n \leq a-1 \end{aligned}$$

Assuming the steady state probabilities of P_{nR} , Q_{nW} and R_{nI} exist and denoted as the analyser is in regular busy period, in working vacation period and in idle period respectively. Then

$$P_{nR} = \lim_{t \rightarrow \infty} P_{nR}(t); Q_{nW} = \lim_{t \rightarrow \infty} Q_{nW}(t); R_{nI} = \lim_{t \rightarrow \infty} R_{nI}(t)$$

During working vacation and regular busy period there will be n number of patients samples waiting for testing in the queue and the system has k ($a \leq k \leq b$) samples in service, whereas in idle period the number of samples waiting for testing in the queue and in the system are same.

Let b_k denote the probability that k batches are served at regular service rate as μ_{RB} in an inter arrival time. Then

$$b_k = \int_0^{\infty} e^{-\mu_R t} \frac{(\mu_{RB} t)^k}{k!} dG(t) \quad k \geq 0$$

We define w_k be the probability that working vacation time is greater than an interarrival time and the service completion of k – batches occur at a rate μ'_{WV} in an inter-arrival time. Then

$$w_k = \int_0^{\infty} e^{-\eta t} \frac{(e^{-\mu'_{WV} t})(\mu'_{WV} t)^k}{k!} dG(t) \quad k \geq 0$$

Taking t_k as the probability that the server returns from vacation in an inter arrival time and k service completions occur in an inter arrival time. Then

$$t_k = \int_0^{\infty} \sum_{i=0}^k \left\{ \int_0^t \eta e^{-\eta x} \frac{(\mu'_{WV} x)^i}{i!} e^{-\mu'_{WV} x} \frac{(\mu_{RB} (t-x))^{k-i}}{(k-i)!} e^{-\mu_{RB} (t-x)} dx \right\} dG(t) \quad k \geq 0$$

Suppose d_k indicates the probability that the analyzer gets breakdown at the rate of β_v during working vacation period. Then

$$d_k = \int_0^{\infty} e^{-\beta_v t} \frac{(\beta_v t)^k}{k!} dG(t), \quad k \geq 0$$

Assuming e_k represents the probability that the analyzer gets breakdown at the rate of β_R during regular busy period. Then

$$e_k = \int_0^{\infty} e^{-\beta_R t} \frac{(\beta_R t)^k}{k!} dG(t), \quad k \geq 0$$

By considering the above probabilities steady state equations are written and by applying the displacement between the state of Markov chain the steady state solution at pre - arrival time are calculated.

Steady State Equations:

During Idle Period:

$$R_{nI} = R_{(n-1)I} + \sum_{k=0}^{\infty} Q_{kb+n-1} (1 - \sum_{i=0}^k (w_i + t_i + d_i)) + \sum_{k=0}^{\infty} P_{kb+n-1} (1 - \sum_{i=0}^k (b_i + e_i))$$

$$1 \leq n \leq a-1 \quad (1)$$

$$R_{0I} = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j} (1 - \sum_{i=0}^k (w_i + t_i + d_i)) + \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} P_{(k-1)b+j} (1 - \sum_{i=0}^k (b_i + e_i)) + R_{(a-1)I} (1 - (t_0 + w_0)) \quad (2)$$

During Working Vacation Period:

$$Q_{nW} = \sum_{k=0}^{\infty} Q_{kb+n-1} (w_k + d_k) \quad n \geq 1 \quad (3)$$

$$Q_{0W} = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j} (w_k + d_k) + R_{a-1I} w_0 \quad (4)$$

During Regular Busy Period:

$$P_{nR} = \sum_{k=0}^{\infty} P_{kb+n-1} (b_k + e_k) + \sum_{k=0}^{\infty} Q_{kb+n-1} (t_k) \quad n \geq 1 \quad (5)$$

$$P_{0R} = \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} P_{(k-1)b+j} (b_k + e_k) + \sum_{k=1}^{\infty} \sum_{j=a-1}^{b-1} Q_{(k-1)b+j} (t_k) + R_{a-1I} t_0 \quad (6)$$

With the aid of the steady state equation, steady state solution is derived.

Steady State Solution:

To obtain steady state solution we use forward shifting operator E on P_{nR} and Q_{nW} as

$$E(P_{nR}) = P_{n+1} \text{ and } E(Q_{nW}) = Q_{n+1}$$

Thus, the Eq (3) gives the homogeneous equation:

$$(E - \sum_{k=0}^{\infty} E^{kb} (w_k + d_k)) Q_{nW} = 0, \quad n \geq 0 \quad (7)$$

And the Eq (5) gives the non-homogeneous equation:

$$(E - \sum_{k=0}^{\infty} E^{kb} (b_k + e_k)) P_{nR} = \sum_{k=0}^{\infty} Q_{kb+n} (t_k) \quad n \geq 0 \quad (8)$$

To obtain P_{nR} , Q_{nW} , and R_{nI} , let us assume $B(z^b)$, $W(z^b)$, $D(z^b)$ and $E(z^b)$ respectively to denote Probability Generating Function of b_k , w_k , d_k and e_k then we have:

$$B(z^b) = \sum_{k=0}^{\infty} b_k z^{kb} = G^*(\mu_{RB}(1 - z^b)) \quad (9)$$

$$W(z^b) = \sum_{k=0}^{\infty} w_k z^{kb} = G^*(\eta + \mu'_{wv}(1 - z^b)) \quad (10)$$

$$D(z^b) = \sum_{k=0}^{\infty} d_k z^{kb} = G^*(\eta + \beta_v(1 - z^b)) \quad (11)$$

$$E(z^b) = \sum_{k=0}^{\infty} e_k z^{kb} = G^*(\beta_R(1 - z^b)) \quad (12)$$

If $\rho = (\frac{\lambda}{b\mu}) < 1$, then the equation $z = B(z^b)$ has a unique root r_R inside (0,1). This result follows from the corresponding result of GI/M/1 model of Gross and Harris (1985), $B(r_R^b) = r_R$ with $0 < r_R < 1$.

If $\eta > 0$ then the equation $G^*(\eta + \mu'_{wv}(1 - z^b))$ has a unique root r_{1V} in the interval (0,1), (i.e) $W(r_{1V}^b) = r_{1V}$ with $0 < r_{1V} < 1$.

If β_v and β_R are in (0,1), then the equation $D(z^b) = z$ and $E(z^b) = z$ have unique roots such as $r_{1\beta v}$ and $r_{\beta R}$ lie inside (0,1) respectively.

Hence the homogeneous difference Eq (7) has a solution

$$Q_{nW} = r_{1V}^n Q_{0W} \quad n \geq 0 \quad (13)$$

And the non-homogeneous difference Eq (8) has a solution

$$P_{nR} = (Lr_R^n + M r_{1V}^n) Q_{0W} \quad n \geq 0 \quad (14)$$

$$\text{Where } M = \frac{T(r_{1V}^b)}{r_{1V} - B(r_{1V}^b) - D(r_{1V}^b) - F(r_{1V}^b)}$$

$$\text{and } Tr_{1V}^b = \sum_{k=0}^{\infty} r_{1V}^{kb} t_k = M(r_{1V} - B(r_{1V}^b) - D(r_{1V}^b) - F(r_{1V}^b)) \quad (15)$$

$$\text{Thus } P_{nR} = \left[Lr_R^n + \left(\sum_{k=0}^{\infty} r_{1V}^{kb} t_k \right) \frac{r_{1V}^n}{r_{1V} - B(r_{1V}^b) - D(r_{1V}^b) - F(r_{1V}^b)} \right] Q_{0W} \quad \text{where } r_{1V} \neq r_R \quad (16)$$

The expression R_{nI} is obtained by substituting Q_{nW} and P_{nR} from Eq (13) & (14)

$$R_{nI} = \left[L \left(\frac{1-r_R-\beta_R}{1-r_R} \right) \left(\frac{r_R^{a-1}-r_R^n}{1-r_R^b} \right) + (M+1) \left(\left(\frac{1-r_{1V}-\beta_v}{1-r_{1V}} \right) \left(\frac{r_{1V}^{a-1}-r_{1V}^n}{1-r_{1V}^b} \right) \right) + \frac{r_{1V}^{a-1}-r_{1V}^n}{r_{1V}^b(1-r_{1V})} + \frac{r_{1V}^b-r_{1V}^a}{w_0 r_{1V}^b(1-r_{1V})} \right] Q_{0W}$$

$$0 \leq n \leq a-1 \quad (17)$$

Thus, the steady state queue size probabilities are expressed in terms of Q_{0W} and L .

Now L can be determined by substituting Q_{nW} and P_{nR} in Eq (6):

Thus

$$L \left(\frac{r_R^a - r_R^b + (b_0)(r_R^b - r_R^{a-1})}{r_R^b(1-r_R)} \right) = \frac{t_0}{w_0} \left(\frac{r_{1V}^a - r_{1V}^b}{r_{1V}^b(1-r_{1V})} \right) + \beta_v \left(\frac{r_{1V}^{a-1} - r_{1V}^b}{r_{1V}^b(1-r_{1V})} \right) - M \left(\frac{r_{1V}^a - r_{1V}^b + (b_0)(r_{1V}^b - r_{1V}^{a-1})}{r_{1V}^b(1-r_{1V})} \right) \quad (18)$$

Hence the steady state queue size probabilities of described model are expressed in terms of Q_{0W} then:

$$Q_{nW} = r_{1V}^n Q_{0W} \quad n \geq 0 \quad (19)$$

$$P_{nR} = (Lr_R^n + M r_{1V}^n) Q_{0W} \quad n \geq 0 \quad (20)$$

$$R_{nI} = \left[L \left(\frac{1-r_R-\beta_R}{1-r_R} \right) \left(\frac{r_R^{a-1}-r_R^n}{1-r_R^b} \right) + (M+1) \left(\left(\frac{1-r_{1V}-\beta_v}{1-r_{1V}} \right) \left(\frac{r_{1V}^{a-1}-r_{1V}^n}{1-r_{1V}^b} \right) \right) + \frac{r_{1V}^{a-1}-r_{1V}^n}{r_{1V}^b(1-r_{1V})} + \frac{r_{1V}^b-r_{1V}^a}{w_0 r_{1V}^b(1-r_{1V})} \right] Q_{0W}$$

$$0 \leq n \leq a-1 \quad (21)$$

where;

$$L = \left(\frac{r_R^b(1-r_R)}{r_R^a - r_R^b + (b_0)(r_R^b - r_R^{a-1})} \right) \left[\frac{t_0}{w_0} \left(\frac{r_{1V}^a - r_{1V}^b}{r_{1V}^b(1-r_{1V})} \right) + \beta_v \left(\frac{r_{1V}^{a-1} - r_{1V}^b}{r_{1V}^b(1-r_{1V})} \right) - M \left(\frac{r_{1V}^a - r_{1V}^b + (b_0)(r_{1V}^b - r_{1V}^{a-1})}{r_{1V}^b(1-r_{1V})} \right) \right]$$

$$M = \frac{\sum_{k=0}^{\infty} r_{1V}^{kb} t_k}{\beta_v} = \frac{\lambda \eta}{\beta_v \left(\frac{\mu_R}{\lambda + \mu} - \frac{\mu'_{wv}}{\lambda + \mu'_{wv} + \eta} \right)} \left(\frac{\mu_R}{\lambda + \mu_R - r_{1V}^b \mu_R} - \frac{\mu'_{wv}}{\lambda + \mu_R + \eta - r_{1V}^b \mu'_{wv}} \right)$$

$$t_0 = \frac{\eta}{\lambda + \mu_R}; \quad w_0 = \frac{\lambda}{\lambda + \eta + \mu'_{wv}}; \quad b_0 = \frac{\lambda}{\lambda + \mu_R}$$

By using normalizing conditions, the value of Q_{0W} is calculated.

$$\text{(i.e.) } \sum_{n=0}^{\infty} P_{nR} + \sum_{n=0}^{\infty} Q_{nW} + \sum_{n=0}^{\infty} R_{nI} = 1$$

$$\text{Then } Q_{0W}^{-1} = L(h(r)) + (R+1)h(r_1) + \frac{1}{r_{1V}^b(1-r_{1V})} \left(\frac{r_{1V}^b - r_{1V}^a}{w_0} + (r_{1V}^{a-1} - r_{1V}^b) \right) \quad (22)$$

Thus, the steady state queue size probabilities for the general arrival bulk service with types break down are given by

$$Q_{nW} = r_{1V}^n Q_{0W} \quad n \geq 0$$

$$P_{nR} = (Lr_R^n + M r_{1V}^n) Q_{0W} \quad n \geq 0$$

$$R_{nl} = \left[L \left(\frac{1-r_R-\beta_R}{1-r_R} \right) \left(\frac{r_R^{a-1}-r_R^n}{1-r_R^b} \right) + (M+1) \left(\left(\frac{1-r_{1V}-\beta_V}{1-r_{1V}} \right) \left(\frac{r_{1V}^{a-1}-r_{1V}^n}{1-r_{1V}^b} \right) \right) + \frac{r_{1V}^{a-1}-r_{1V}^n}{r_{1V}^b(1-r_{1V})} + \frac{r_{1V}^b-r_{1V}^a}{w_0 r_{1V}^b(1-r_{1V})} \right] Q_{0W} \quad 0 \leq n \leq a-1$$

where Q_{0W} is determined using Eq 22.

Performance measures:

Mean Queue Length:

The mean queue length L_q of the model is as follows:

$$L_q = \sum_{n=0}^{\infty} n P_{nR} + \sum_{n=0}^{\infty} n Q_{nW} + \sum_{n=0}^{a-1} n R_{nl}$$

By substituting P_{nR} , Q_{nW} and R_{nl} we get:

$$L_q = \left[L(\xi(r)) + (M+1)(\xi(r_1)) + \frac{a(a-1)}{2} \left(\frac{r_{1V}^{a-1}-r_{1V}^b}{r_{1V}^b(1-r_{1V})} + \frac{r_{1V}^b-r_{1V}^a}{w_0 r_{1V}^b(1-r_{1V})} \right) \right] Q_{0W} \quad (23)$$

$$\text{where } \xi(x) = \frac{x}{(1-x)^2} + \frac{1-x-\beta_R}{(1-x)(1-x^b)} \left(\frac{a(a-1)x^{a-1}}{2} + \frac{ax^a(1-x)-x(1-x^a)}{(1-x)^2} \right) \text{ and } w_0 = \frac{\lambda}{\lambda+\eta+\mu'_{WV}}$$

Thus the mean queue length of GI/M(a,b)/1/ MWV with types of breakdown is derived.

Sensitivity Analysis

To evaluate the robustness and efficiency of queueing systems under varying operational conditions sensitivity analysis is a powerful tool. In this study we analyse the sensitivity of GI/M(a,b)/1 with multiple working vacations, where both regular and working vacation periods are subject to distinct break down mechanism. The arrival process follows general distribution with rate λ , while service time follows exponential distribution with rate μ_R during regular busy periods and μ'_{WV} during working vacations. Server returns from the working vacation within an interarrival time is governed by the parameter η . Breakdown occurrences are modeled by parameters β_v and β_R corresponding to the failures during working vacation and regular busy periods respectively. The testing is performed in batches of size $a = 5, 6, 7, 8, 9, 10$ upto a maximum batch size $b = 12$. Through numerical analysis, we examined how variations in the breakdown parameters influence the mean queue length. This analysis provides valuable insights into how the reliability of the analyser during working vacation and regular busy periods affects overall system congestion. The results assist in identifying optimal diagnostic laboratory configurations that results in maintaining the effective performance while minimizing the impact of breakdowns.

Thus, for various values of β_v and β_R , the mean queue length L_q are tabulated and the values are shown in the Fig.1. By letting $\lambda = 0.5$, $\mu_R = 0.7$, $\mu'_{WV} = 0.4$ and $\eta = 0.2$ and by varying batch size of “a” from 5 to 12 the values of L_q are calculated and tabulated in Table 1.

Table 1: Mean queue length for GI/M(a,b)/1/MWV with respect to β_v and β_R

β_R β_v	0.01	0.02	0.03	0.04	0.05
0.005	24.87939	25.97134	28.06328	29.15523	29.52472
0.006	40.16934	43.52708	43.72258	44.87372	47.51783
0.007	62.76327	65.24755	66.86245	67.47735	68.09225
0.008	83.15424	83.38954	84.22483	86.36013	89.49542
0.009	102.5167	105.5337	107.6073	109.8677	112.8847

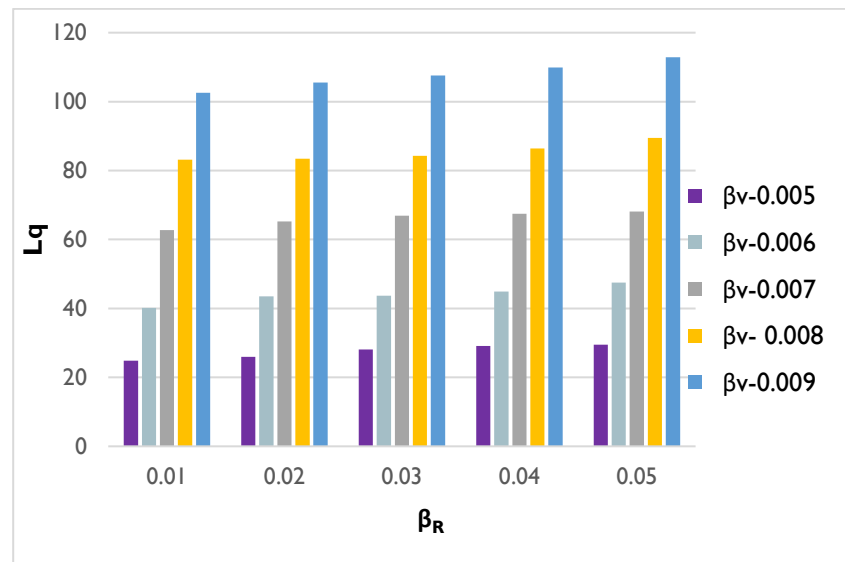


Fig. 1. L_q with respect to β_v and β_R

The Fig. 1. has been plotted for various values of β_v – break down during working vacation ranging from 0.005 to 0.009 and for the values of β_R – breakdown during regular busy period ranging from 0.01 to 0.05. From the graph it is observed that for fixed value of β_R , the bar height increases as β_v increases. This indicates that the queue length increases when the β_v is higher even though β_R is fixed. As β_R increases across bar groups (left to right) all bars in each group grows taller, shows that β_R has a strong impact on the patient samples waiting for testing in the queue. This is because higher breakdowns in regular busy periods directly reduce the effective service rate, and causing increase in queue length. From this one can conclude that the patient samples waiting for testing in the queue is positively correlated with breakdown during the regular busy period and the impact becomes more significant as the breakdown during working vacation increases. The graph depicts that the effective system performance depends on minimizing both types of breakdowns with special focus on regular busy period maintenance.

RESULTS AND DISCUSSION

The prescribed model incorporates General arrival under multiple working vacations with minimum batch size a from 5 to 10 and maximum batch size b is 12. In this model steady state equations and expected queue length is calculated for various breakdown parameter β_v and β_R during working vacation and regular busy period respectively. From the results it shows that increasing the breakdown rate shows a significant increase in expected samples waiting for testing in the queue. Finally, the finding highlight how analyser failures in both busy and vacation modes significantly affects diagnostic laboratory performance.

CONCLUSION

This paper analyzes a comprehensive analysis of the GI/M(a,b)/1 with multiple working vacation with types of breakdowns occurring during working vacation and regular busy periods. Constructing and solving the embedded Markov chain at service completion at pre-arrival epochs steady state probabilities has derived. Performance measure such as expected patients' samples waiting for testing in the queue length is calculated. A sensitivity analysis has been examined to note the impact of break down probabilities on diagnostic laboratory performance. Graphical representation of bar graph in 2D plots demonstrates that how breakdown parameters affect the entire diagnostic laboratory process queue length. In future the model can be extended to multi-server laboratory environments to enhance the quality of laboratory.

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