Mathematical Model of Magnetic Influence on Oxygen and Nutrient Delivery in Arteries with Stenosis and Thrombosis

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ABSTRACT

The current study creates a mathematical model to examine oscillatory arterial blood flow in the presence of minor stenosis using a magnetic field. To better understand the irregular flow behavior of blood in a locally constricted artery, analytical findings were obtained to investigate the role of magnetic effect on oscillatory blood flow, which was modeled as a Newtonian fluid. The arterial wall surface roughness is supposed to have a cosine-shaped profile, with a tiny maximum amplitude as compared to the radius of the unblocked artery.

The fundamental goal of this study is to construct an acceptable mathematical model to examine the effect of a magnetic field on oscillatory arterial blood flow in the presence of stenosis. A mathematical model has been developed to study the magnetic effect on oxygen and nutrition delivery in arteries with modest stenosis and thrombosis. The solutions provide insights into flow characteristics and hemodynamic factors, such as wall shear stress and velocity profiles.

KEYWORDS: Mathematical model, oscillatory arterial blood flow, Newtonian fluid, magnetic effects, Stenosis.

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INTRODUCTION

The study of blood flow hemodynamics sheds light on the mechanisms underlying cardiovascular disorders such as plaque formation and coronary artery disease. Stenosis is the construction of an artery, which is frequently caused by the buildup of fatty deposits known as atherosclerosis plaques. Atherosclerosis, which is defined by the accumulation of lipid molecules within the artery wall, reduces blood flow through the afflicted vessel. The major effect of stenosis is a reduction in blood flow, which results in an insufficient supply of oxygen and nutrients to surrounding tissues and organs. Furthermore, the irregular surface of a plaque promotes blood clot (thrombus) formation within the stenotic region. A dislodged clot can travel through the bloodstream and obstruct smaller vessels, resulting in severe clinical complications. Blood is made up of three types of cells: red blood cells, white blood cells, and platelets, all suspended in plasma. While plasma is a Newtonian fluid, entire blood behaves differently at low shear rates due to interactions between its cellular components. Blood, on the other hand, can be approximated as a Newtonian fluid at higher shear rates, which are common in bigger arteries. Blood flow across stenosed arteries has received a lot of attention in studies because of its physiological importance and clinical implications. The term stenosis refers to a variety of blood vessel problems and is a major contemporary health concern. Stenosis is a partial or total obstruction of the artery lumen caused by fatty material deposition, resulting in reduced blood flow. It is one of the most common arterial disorders of the circulatory system. Numerous scholars have built theoretical and practical models to study vascular stenosis. Blood flow via an artery is primarily controlled by the heart's pumping motion, which creates a pressure gradient and oscillatory flow within the blood vessel. Cardiovascular disease refers to a wide range of problems affecting the heart and blood arteries, including coronary artery disease, stroke, and heart failure. It is the largest cause of death worldwide, often caused by the development of plaque in arteries known as atherosclerosis. While symptoms are not always present, common signs include chest pain, shortness of breath, nausea, and exhaustion; if symptoms arise, rapid medical attention is required. A healthy lifestyle includes regular exercise, a balanced diet low in salt and unhealthy fats, quitting smoking, and controlling other health concerns such as high blood pressure and diabetes.
Womersley (1955) investigated the oscillatory motion of a viscous fluid in a rigid tube using a simple harmonic pressure gradient and the effect of frequency on the instantaneous flow rate. Barnes et al. (1971) explored the non-Newtonian flow behavior of fluids through straight, rigid circular tubes, finding remarkable agreement between theoretical and experimental results.

Daly (1976) investigated pulsatile flow through canine femoral arteries with lumen constriction numerically, whereas Back et al.

(1977) numerically solved the governing equations of fluid dynamics to examine pulsatile blood flow through coronary arteries containing multiple non-obstructive plaques. Newman et al. (1979) numerically analysed oscillatory flow in a rigid tube with stenosis, whereas Doffin and Chagneau (1981) investigated oscillating flow between a stenosis and a clot. Kumar et al. (2005) investigated computational methods for studying blood flow with porous effects in arterial arteries. Bhardwaj and Kanodia (2007) conducted comprehensive research on oscillatory arterial blood flow in the presence of mild stenosis, whereas Rathod and Tanveer (2009) examined the pulsatile flow of a couple-stress fluid through a porous media under periodic body acceleration and magnetic field effects. Furthermore, Sanyal and Biswas (2010) investigated the pulsatile blood flow through an axisymmetric artery in the presence of a magnetic field.

The current study builds on the work of Bhardwaj and Kanodia (2007) by including the influence of a porous medium into the analysis of oscillatory blood flow through a stenosis artery. The model's current work combines magneto hydrodynamics (MHD) and fluid dynamics equations. It describes blood flow using the Navier-Stokes and continuity equations, which are adjusted to account for magnetic body forces as well as the geometric consequences of stenosis and clot formation. The governing equations are solved analytically or numerically to estimate the effect of magnetic fields on blood velocity, pressure distribution, and nutrition and oxygen delivery.

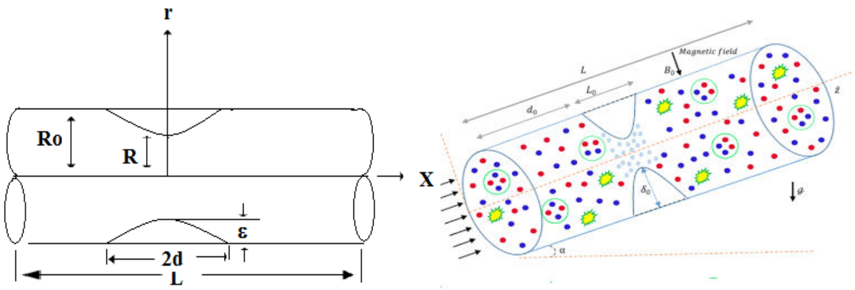
To imitate the pulsatile nature of cardiac motion, arterial blood is considered to behave as a Newtonian fluid with a periodic pressure gradient driving the flow. The model accounts for the impacts of artery constriction geometry, magnetic body forces, and oscillatory flow properties. The governing Navier-Stokes and continuity equations, modified to include magnetic effects, are constructed and solved under appropriate boundary conditions to investigate the fluctuations in velocity, pressure, and wall shear stress inside the stenotic artery segment.

FORMULATION OF PROBLEM

The mathematical model used to assess oscillatory arterial blood flow in the presence of stenosis under the influence of a magnetic field is described in this communication. It is assumed that the flow is irregular and axially symmetric. It is assumed that density and viscosity are constants. The artery has a consistent radius both before and after the stenosis. Because of any aberrant increase in the artery's lumen, it is frequently considered that constriction occurs symmetrically. Figure 1 shows the idealized geometry of stenosis.

$$\frac{R(x)}{R_0} = 1 - \frac{\varepsilon}{2R_0}\left[1 + \cos\frac{\pi x}{d}\right] \tag{1}$$

where R_0 is the radius of the normal artery, R(x) is the radius of the artery in the stenotic region, 2d is the length of stenosis and h is the maximum height of the stenosis such that $\varepsilon / R_0 << 1$.



(a) **(b)**

Fig.1 Geometrical Structure of Stenosis arteries

The equation of motion governing the flow field in the tube is

$$\rho\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 w}{\partial t^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - Mw \tag{2}$$

where M is Hartmann number, p is the fluid pressure, ρ is the density and w is the velocity in the axial direction, μ is the viscosity.

The boundary conditions are

$$w = 0 \text{ on } r = R \tag{3}$$

$$\frac{\partial w}{\partial r} = 0 \text{ on } r = 0 \tag{4}$$

ANALYTICAL SOLUTION OF MATHEMATICAL MODEL

The simple solution to the motion of a viscous fluid is achieved in the section under the changing pressure gradient. Before proceeding with the solution, the transformation specified by y = r/R_0 is accomplished.

The basic equation (2) becomes after employing the boundary conditions (3) and (4) as shown below.

$$\frac{\partial^2 w}{\partial y^2} + \frac{1}{y}\frac{\partial w}{\partial y} - \frac{\rho R_o^2}{\mu}\frac{\partial w}{\partial t} - \left(\frac{R_o^2}{M}\right) w = \frac{R_o^2}{\mu}\frac{\partial p}{\partial x} \tag{5}$$

$$w = 0 \text{ on } y = \frac{R}{R_0}$$

$$\frac{\partial w}{\partial y} = 0 \text{ on } y = 0 \tag{6}$$

Set the solution to P and W in the for.

$$w(y,t) = W(y)e^{i\omega t} \quad -\frac{\partial p}{\partial x} = Pe^{i\omega t} \tag{7}$$

From (7) into equation (5) we get

$$\frac{d^2 W}{dy^2} + \frac{1}{y}\frac{dW}{dy} - \beta^2 W = -\frac{{R_0}^2}{\mu}P \tag{8}$$

where

$$\beta^2 = \frac{i\rho R_o^2 \omega}{\mu} + \frac{R_o^2}{k} \tag{9}$$

The solution to equation (8) under the boundary condition (6) is given by $W(y) = \frac{P{R^2}_0}{\mu\beta^2}\left[1 - \frac{B_0(\beta y)}{B_0(\beta \frac{R}{R_0})}\right]$ (10)

where B_0 is the Bessel function of order zero.

Then the expression for the axial velocity in the tube is given by $w(r,t) = \frac{P{R_0}^2}{\mu\beta^2}\left[1 - \frac{B_0\left(\beta\frac{r}{R_0}\right)}{B_0\left(\beta\frac{R}{R_0}\right)}\right].e^{iwt}$ (11)

The volumetric flow rate Q as given below

$$Q = 2\pi\int_0^R w\,r\,dr \tag{12}$$

which gives on integration, at once

$$Q = \frac{\pi P{R_0}^4}{\mu\beta^2}\left(\frac{R}{R_0}\right)\left[\frac{R}{R_0} - \frac{2B_1\left(\beta\frac{R}{R_0}\right)}{B_0\left(\beta\frac{R}{R_0}\right)}\right].e^{iwt} \tag{13}$$

$$\frac{\partial p}{\partial x} = -Q\mu\beta^2 \left[\frac{R}{R_0} - \frac{2B_1\left(\beta\frac{R}{R_0}\right)}{B_0\left(\beta\frac{R}{R_0}\right)} \right]^{-1} \cdot \frac{1}{\pi R_0{}^4\left(\frac{R}{R_0}\right)} \tag{14}$$

Where B_1 Bessel's function of first kind.
The shear stress at the wall r = R is given by

$$\tau_R = \mu\left(\frac{\partial w}{\partial r}\right)_{r=R} \tag{15}$$

Putting (11) for w into above equation and using equation (13) for Q, one obtains τ_R as given below

$$\frac{\tau_R}{Q} = \frac{-\mu\beta^2 B_1\left(\beta\frac{R}{R_0}\right)}{\pi R_0{}^3\left[\beta\left(\frac{R}{R_0}\right)^2 B_0\left(\beta\frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right)B_1\left(\beta\frac{R}{R_0}\right)\right]} \tag{16}$$

The τ_R is calculated by the below formula

$$\tau_N = -\frac{R_0}{2}\left(\frac{\partial p}{\partial x}\right)_0 \tag{17}$$

The shear stress is given by

$$|\bar{\tau}| = \left|\frac{\tau_R}{\tau_n}\right| = \frac{1}{-\frac{R_0}{2}\left(\frac{\partial p}{\partial x}\right)_0} \frac{R\left(\frac{\partial p}{\partial x}\right)_0 B_1\left(\beta\frac{R}{R_0}\right)\left\{\frac{R}{R_0} - \frac{2B_1\left(\beta\frac{R}{R_0}\right)}{B_0\left(\beta\frac{R}{R_0}\right)}\right\}}{\left[\beta\left(\frac{R}{R_0}\right)^2 B_0\left(\beta\frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right)B_1\left(\beta\frac{R}{R_0}\right)\right]} \tag{18}$$

$$|\bar{\tau}| = -2\left(\frac{R}{R_0}\right) \frac{J_1\left(\beta\frac{R}{R_0}\right)\left\{\frac{R}{R_0} - \frac{2J_1\left(\beta\frac{R}{R_0}\right)}{J_0\left(\beta\frac{R}{R_0}\right)}\right\}}{\left[\beta\left(\frac{R}{R_0}\right)^2 J_0\left(\beta\frac{R}{R_0}\right) - 2\left(\frac{R}{R_0}\right)J_1\left(\beta\frac{R}{R_0}\right)\right]} \tag{19}$$

The resistance impedance to the flow as given below:

$$Z = -\left(\frac{\partial p/\partial x}{Q}\right) \tag{20}$$

RESULTS AND DISCUSSION

From figures 2 and 3, it is observed that all axial velocity graphs decrease to r = 1, then each graph begins to decrease after velocity and tends to zero with the increase in y. It is also observed from Fig.2 and 3 that with the increase in M and R/R_o, velocity increases, i.e. blood flow increases in the artery stenosis region. Fig 4 and fig 5 depicts the average velocity versus time of various value of Hartmann number.

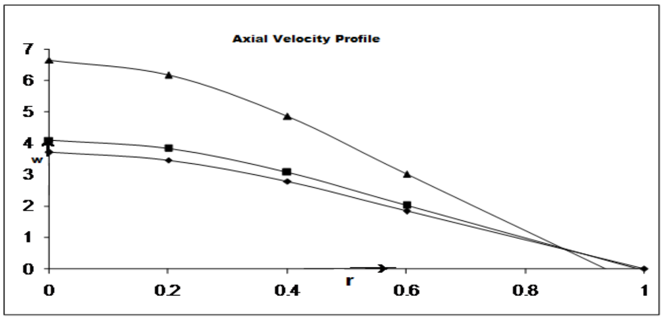


Figure 2. Various value M of axial velocity profile.

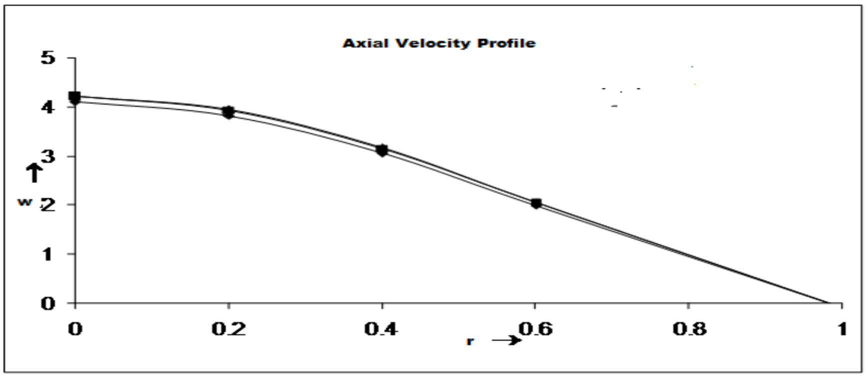


Figure 3 Various value M of axial velocity profile

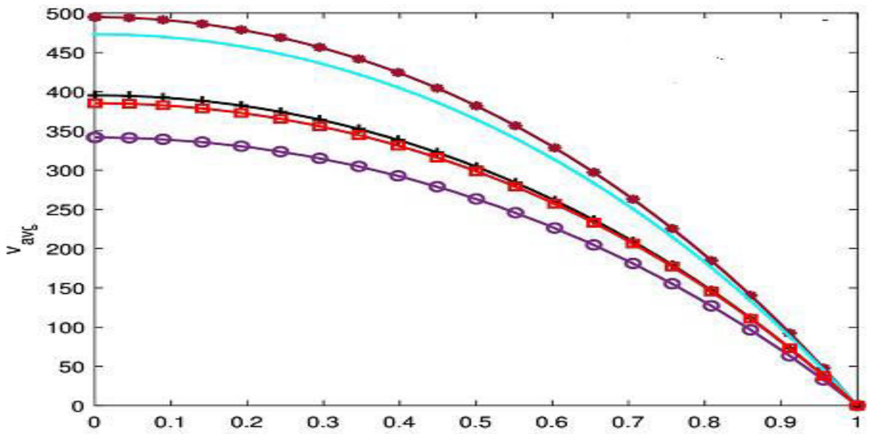


Figure 4. Average velocity vs pressure M=1.2, 1.5 and 2.0

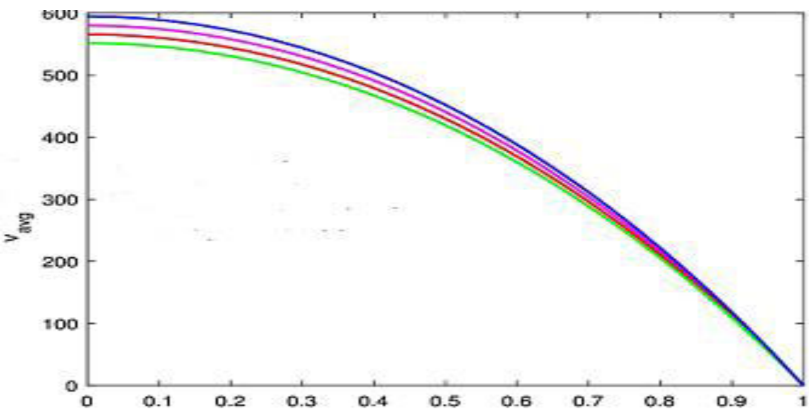


Figure 5. Average velocity vs time M=1.2, 1.5 and 2.0.

CONCLUSION:

The current study creates a mathematical model to analyze oscillatory arterial blood flow through a slightly stenosis artery under the influence of magnetic field. The application of a magnetic field produces an induced field and a force that opposes the direction of blood flow through the stenosis artery model. This magnetic force impacts the velocity distribution within the vessel, which affects the movement of oxygen and nutrients. The mathematical model includes parameters such as the Hartmann number, which measures the relative strength of the magnetic field's influence on flow dynamics. The cardiovascular disease refers to a wide range of illnesses that affect the heart and blood arteries. It is typically related with the accumulation of fatty deposits within the arteries (atherosclerosis) and an increased risk of blood clots. The most major behavioral risk factors for heart disease and stroke include a poor diet, physical inactivity, tobacco use, and problematic alcohol consumption. Air pollution is one of the most significant environmental risk factors.

REFERENCES

1. Barnes, H.A.; Townsend, P. and Walters, K. (1971): On Pulsatile Flow of non-Newtonian Liquids, Rheo. Acta., 10, 517.
2. Back, L.D.; Radbill, J.R. and Crawford, D.W. (1977) : Analysis of Pulsatile, Viscous Blood Flow through Diseased Coronary Arteries of Man, Journal of Biomechanics, 10, 339-155.
3. Bhardwaj, Kavita and Kanodia, K.K. (2007) : Oscillatory Arterial Blood Flow With Mild Stenosis, Acta Ciencia Indica, XXXIII, No. 1, 71.
4. Daly, B.J. (1976): A Numerical Study of Pulsatile Flow through Stenosed Canine Femoral Arteries, Journal of Biomechanics, 9, 465.-478.
5. Newman, D.L., Westerhof, N. and Sipkema, P.: Modeling of Aortic Stenosis, J. Biomechanics, 12, 229 (1979).
6. Rathod, V.P. and Tanveer, S. (2009): Pulsatile Flow of Couple Stress Fluid through a Porous Medium with Periodic Body Acceleration and Magnetic Field, Bull. Malays. Math. Sci. Soc. (2), 32(2), 245-259.
7. Sanyal, D.C. and Biswas, A (2010): Pulsatile Motion of Blood through an Axi-Symmetric Artery in Presence of Magnetic Field, Assam University, Journal of Science and Technology: Physical Sciences and Technology, vol. 5 (II) 12-20.
8. Womersley, J.R. (1955) : Oscillatory Motions of a Viscous Liquids in a Thin-walled Elastic Tube, Phil. Mag., 46, 199-205.
9. Anil Kumar, CL Varshney and GC Sharma (2005): Computational technique for flow in blood vessels with porous effects, Applied Mathematics and Mechanics, China, 26(1) 63-72.
10. Anil Kumar, R B Singh, J. Sinha and Akash (2025): Mathematical Modelling and Theoretical Exploration of Hybrid Blood-Nano-fluid Flow in a Stenosed Artery with Magnetization Effect, Journal of Applied Science and Education, vol. 5(2), 1-11.
11. Anil Kumar, Akash, R. B. Singh , A. Murarka (2025): Numerical Investigation of permeability effects on Blood flow in a Stenosed artery using the Crank-Nicolson method, International Journal of Environmental Sciences, vol. 11(17), 1022-1030.
12. Anil Kumar, G. Varshney, and Akash (2025): Modeling and Analysis of Blood Movement in Stenosis vessels under the Stimulus of Permeability and Magnetization Parameters, Utilitas Mathematica, vol. 122, 698-705.
13. Anil Kumar (2021): Mathematical model for porous influence through mild stenosis thrombosis over delivery of oxygen and nutrients arteries, Advances in Mathematics: Scientific Journal vol 10(1) 463-469.
14. Anil Kumar Gupta (2011): Performance Model and Analysis of Blood Flow in Small Vessels with Magnetic Effects, International Journal of Engineering, IJE Transactions A: Basics, vol. 25 (2), 190 -196.
15. R. K. Saket and Anil Kumar (2008): Reliability of convective diffusion process in porous blood vessels, Chemical Product and Process Modeling, Vol.(3) Article (25), 1-17, Canada.
16. Anil Kumar (2010): Computational Model of Blood Flow in the Presence of Atherosclerosis, International Federation for Medical and Biological Engineering, Conference 6th World Congress on Biomechanics, IFMBE Proceedings 31, Springer Publishing, 1591–1594.