Optimal Purchase Quantity and Credit Period for Deteriorating Items with Credit-Dependent Trended Demand

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ABSTRACT

In order to boost demand and draw in additional purchasers, the seller extends the credit time for account settlement. However, the seller is at danger of defaults due to this financing offer. In this study, the seller is the decision-maker, with demand increasing over time and dependent on the permissible credit period. Products in the inventory deteriorate at a constant rate. Default risk in incurring in sales revenue is considered in the profit maximization objective. The study discusses the necessary condition for determining the optimal credit period and purchase quantity and outlines the procedure to obtain the optimal solution. A numerical example is provided to illustrate the results, and a sensitivity analysis is performed to derive managerial insights.

KEYWORDS: Trade Credit, Inventory, Trended Demand, Default Risk, Deterioration.

How to Cite: Digeshkumar B. Shah, Dushyantkumar G. Patel, Zalak A. Patel, Krunal H. Shah, Atulkumar B. Patel, Vimal B. Patel and Ganpatsinh A. Rathva, (2024) Optimal Purchase Quantity and Credit Period for Deteriorating Items with Credit-Dependent Trended Demand, Vascular and Endovascular Review, Vol.7, No.2, 422-429

INTRODUCTION

Harris-Wilson's Economic Order Quantity (EOQ) model operates under the idea that a seller adopts a cash-on-delivery policy. To stay competitive in the marketplace, the seller extends a grace time to buyers, allowing them time to settle outstanding balances without immediate payment. In this window, no interest is typically charged, provided the buyer clears the debt. This policy offers the seller a dual advantage: it entices buyers and serves as an alternative to offering price discounts, thereby preventing competitors from lowering their prices. Though, on the other hand, the offer of delay period leads to default risk for the seller when the buyer declares his inability to pay off. Notably, the longer the grace period, the higher the risk of non-payment.

Goyal (1985) introduced an economic order quantity model tailored for the buyer, assuming the seller presents a fixed credit period. Since then, the concept of credit periods and their variations has been explored by several researchers. Shah *et al.* (2010) published a review article on inventory modeling with trade credits, where all the referenced studies focus primarily on the buyer's perspective when the seller offers a fixed credit period.

Jaggi et al.(2008) presented a model for determining the optimal policy for retailers, factoring in credit-linked demand under a permissible delay in payments. Building on this, Shah et al. (2009) studied the impact of deterioration within Jaggi et al.(2008)'s framework. Later, Lou and Wang (2013) analysed the seller's strategic decision regarding the establishment of a delayed payment period, assuming a constant and deterministic demand. Across these studies, a common conclusion emerged that the length of trade credit directly improves the demand rate. For further insights, one can refer to the articles of Chern et al. (2013) and Teng and Lou (2012).

Deterioration refers to the decline in quality or value of goods due to decay, damage, spoilage, evaporation, obsolescence, pilferage, or the loss of utility or marginal value (Wee(1993)). Items like food grains, fruits, vegetables, pharmaceuticals, fertilizers, pesticides, and radioactive chemicals are examples of deteriorating products, whose effectiveness diminishes as they are stored over time. The literature on deteriorating inventory lot-size models has been reviewed by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), and Bakker et al. (2012).

Jaggi and Khanna (2011) discussed the impact of permissible payment delays on inventory policies for a retailer dealing with imperfect, deteriorating products. Palanivel and Gowri (2018) developed a model for perishable goods, where demand is influenced by the selling price. Shah *et al*.(2016) investigated an inventory system for decaying items with trapezoidal demand, incorporating preservation technologies aimed at reducing deterioration.

Li *et. al.* (2019) explored a problem involving joint pricing, replenishment, and preservation technology investment for non-instantaneously deteriorating items. The preservation technology in their study influenced the duration of the non-deterioration period and the rate of deterioration. Their formulated model accounted for price-dependent demand, time-varying deterioration, and waiting-time-dependent backlog rates.

Sepehri et al. (2021) created a green inventory model for the production of low-quality, deteriorating goods, analyzing the impact of preservation technologies and carbon-reducing methods on the profitability of the inventory system.

Chung et al. (2023) developed an inventory system that accounts for items with fluctuating values over time in real-world settings. They incorporated a time-varying deterioration rate, linked to the maximum lifespan of the products, designating any items that surpassed their maximum lifetime as scrap and no longer viable.

Kumar *et al*. (2024) introduced reminder costs in trade-credited systems to reduce default risk.
In this paper, we develop an economic order quantity model for the seller that incorporates two key factors: (1) trade-credit-dependent trended demand for deteriorating inventory, and (2) the risk of default associated with offering trade credit. We then establish the necessary conditions to determine the optimal solution, examine how various inventory parameters influence this solution, and derive important managerial insights. An algorithm is provided to determine the seller's optimal decision variables, and the model is validated through a numerical example.

NOTATIONS AND ASSUMPTIONS

The proposed problem is using following notations and assumptions.

Notations

A	Ordering cost per order
C	Purchase cost per unit
P	Selling price per unit ; where $P > C$
h	Holding cost per unit per annum
	Deterioration rate, $0 \le \theta \le 1$
M	Credit period offered by the seller to his buyers (a decision variable)
$R(M,t)$	Time and credit period dependent annual demand rate
$I(t)$	Inventory level at any instant of time t, $0 \le t \le T$
T	Cycle time (a decision variable)
Q	Seller's purchase quantity
$\pi(M,T)$	Seller's profit per unit time

Assumptions

The seller deals with single item.
The items in inventory deteriorate at a constant rate. There is no repair or replacement of deteriorated items during the cycle time.
The replenishment rate is infinite.
Shortages are not allowed. Lead-time is zero or negligible.
In global market, sellers keep selling price constant to bound his retailers.
As argued in Teng and Lou (2012), trade credit is similar to price discount. We consider demand rate to be function of time and credit period as

$$R(M,t) = a(1+bt)M^{\beta} \tag{1}$$

where $a > 0$ is scale demand, $0 \le b < 1$ denotes rate of change of demand with respect to time and $\beta > 0$ is constant.
For seller, default risk increases when longer credit period is offered to the buyer. Here, the rate of default risk giving the credit period M is assumed to be

$$F(M) = 1 - M^{-\gamma} \tag{2}$$

where $\gamma > 0$ is a constant.

Mathematical Model

The seller's inventory is depleting due to increasing demand and offer of credit period. The rate of change of inventory is governed by the differential equation

$$\frac{dI(t)}{dt} = -aM^{\beta}(1+bt) - \theta I(t) \tag{3}$$

with $I(T) = 0$. Then solution of differential equation (3) is

$$I(t) = \frac{-1}{\theta^3}\left(\left(\begin{array}{l}\left(\theta^2(-Tb-1)+\theta b\right)e^{\theta t}\\ +e^{\theta t}\left(\theta^2(1+bt)-\theta b\right)\end{array}\right)e^{-\theta t}M^{\beta}a\right) \tag{4}$$

Initially, the seller has Q units in inventory system i.e.

$$Q = I(0) = \frac{1}{\theta}\left(TbM^{\beta}a\right) \tag{5}$$

The relevant costs per cycle for the seller are

Net revenue after default risk:

$$SR = P\int_0^T R(M,t)\,dt\left(1-F(M)\right)$$

Purchase cost ; $PC = CQ$

Ordering cost ; $OC = A$

Holding cost ; $HC = h\int_0^T I(t)\,dt$

Hence, the seller's annual profit per unit time is

$$\pi(M,T) = \frac{1}{T}\left[SR - PC - OC - HC\right] \tag{6}$$

To maximize annual profit per unit time with respect to credit period and cycle time, the necessary condition is

$$\left.\begin{array}{l}\dfrac{\partial\pi(M,T)}{\partial M} = 0\\ \dfrac{\partial\pi(M,T)}{\partial T} = 0\end{array}\right\} \tag{7}$$

However, the non-linearity of equation (6) and it partial derivatives will not allow us to derive closed form of solution. So, we propose solution procedure as follows:

Step 1: Assign values to the inventory parameters.

Step 2: Solve equations in (7) simultaneously with mathematical software. We have used Maple XIV.

Step 3: Check second order (sufficiency) conditions.

Step 4: Compute profit $\pi(M,T)$ per unit time from equation (6) and purchase quantity Q using equation (5).

In next section, numerical example is given to validate the mathematical model.

Numerical Example and Sensitivity Analysis

Example: Let $a = 1000$ units, $\beta = 3$, $b = 30\%$, $\gamma = 2$, $C = \$10$ per unit, $P = \$25$ per unit, $h = \$0.5$ per unit per annum, $A = \$350$ per order, and $\theta = 50\%$

We find optimal credit period $M = 0.7853$ year and cycle time $T = 1.0933$ years for the supplier. This results in profit of $\$14914.95$ with purchase of 830 units. The concavity of the total profit per unit time is demonstrated in figure 1 w. r. t. M for $T = 1.0933$ and in figure 2 w. r. t. T for $M = 0.7853$. 3-D plot of total profit shows concavity with respect to cycle time T and credit period M in figure 3.

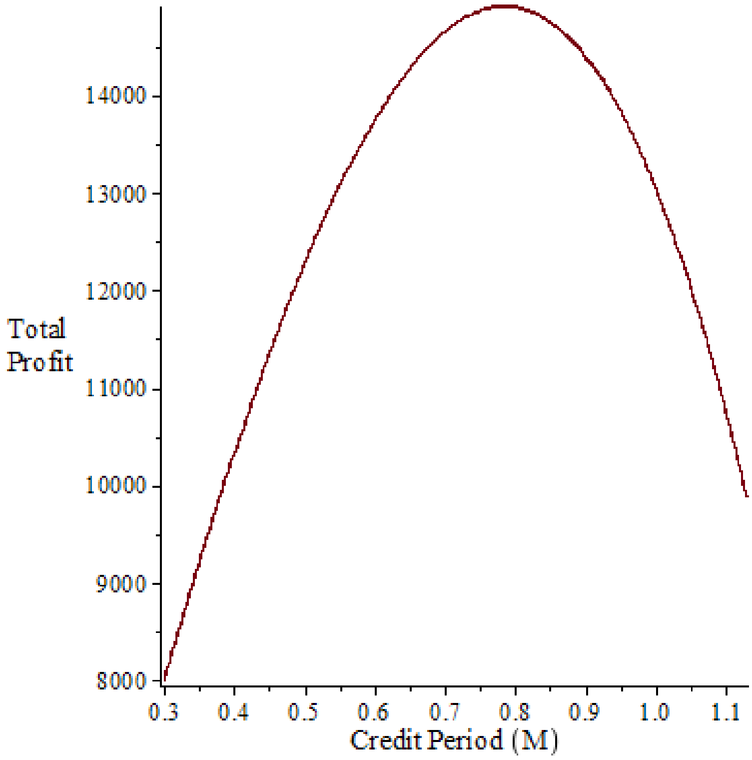

Fig. 1 Concavity of Total Profit w. r. t. Credit Period (*M*)

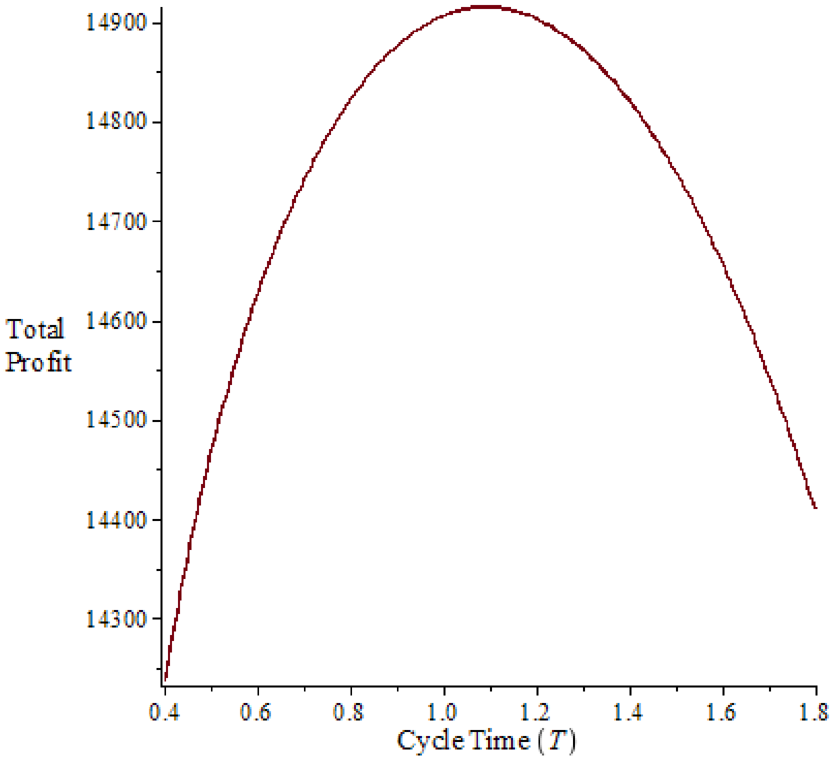

Fig. 2 Concavity of Total Profit w. r. t. Cycle Time (*T*)

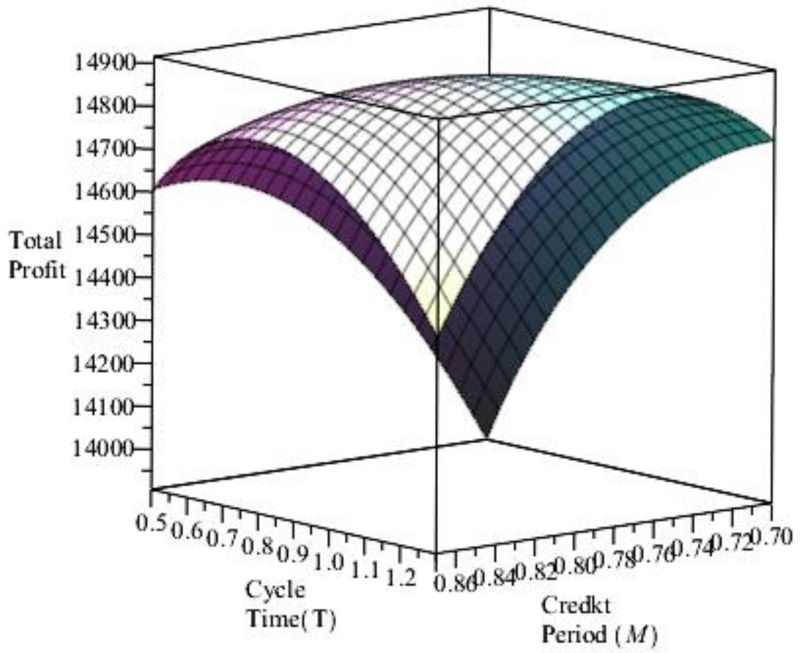

Fig. 3 Concavity of Total Profit w. r. t. Cycle Time (*T*) and Credit Period (*M*)

Now, we study changes in optimal solutions and profit function by altering inventory parameters as -40%,-20%,+20%,+40%; one at time. The deviation in credit period are shown in figure 4, that in cycle time is shown in figure 5, total profit of the supplier is explained in figure 6 and purchase quantity is illustrated in figure 7. It is concluded from fig. 4 that credit period has huge impact of retail price and trade credit elasticity. This validate so long said argument that delay period boost demand. Fig. 5 shows that increase in scale demand, retail price and deterioration decreases cycle time. Ordering cost has large impact on cycle time. Fig 6 reveals that increase in retail price, scale demand and mark up for trade credit have deep impact on total profit and contrast to this increase in trade credit elasticity reduce total profit. Fig 7 explain that purchase quantity get increase with increase in ordering cost, retail price and scale demand whereas with increase in holding cost it decreases. The conduct of the demand with time and delay period is shown in fig. 8.

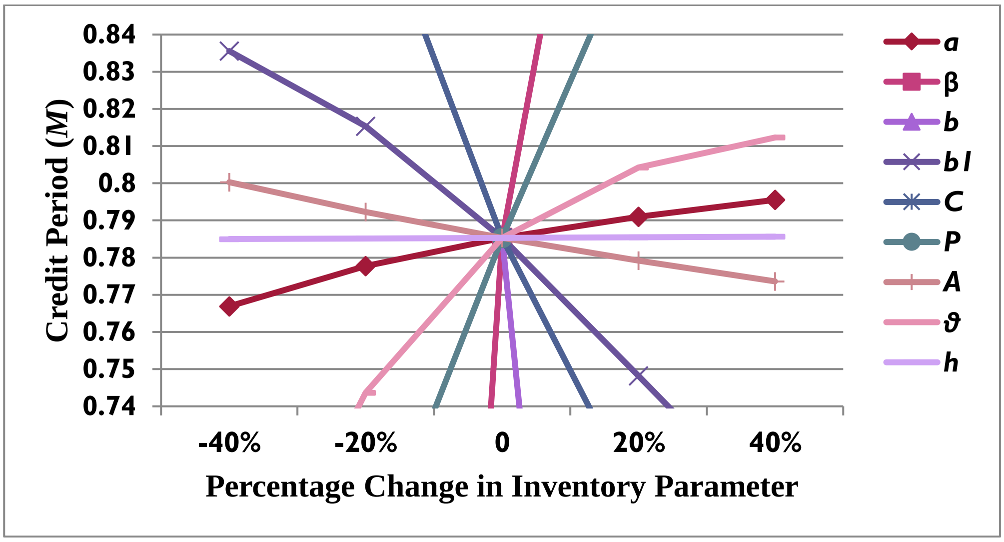

Figure 4 Sensitivity analysis for credit period (*M*)

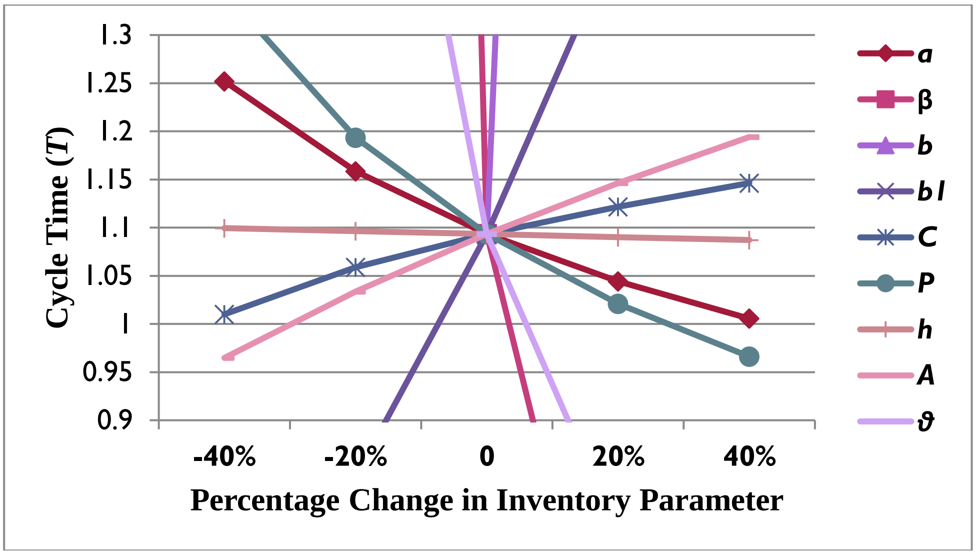

Figure 5 Sensitivity analysis for cycle time (*T*)

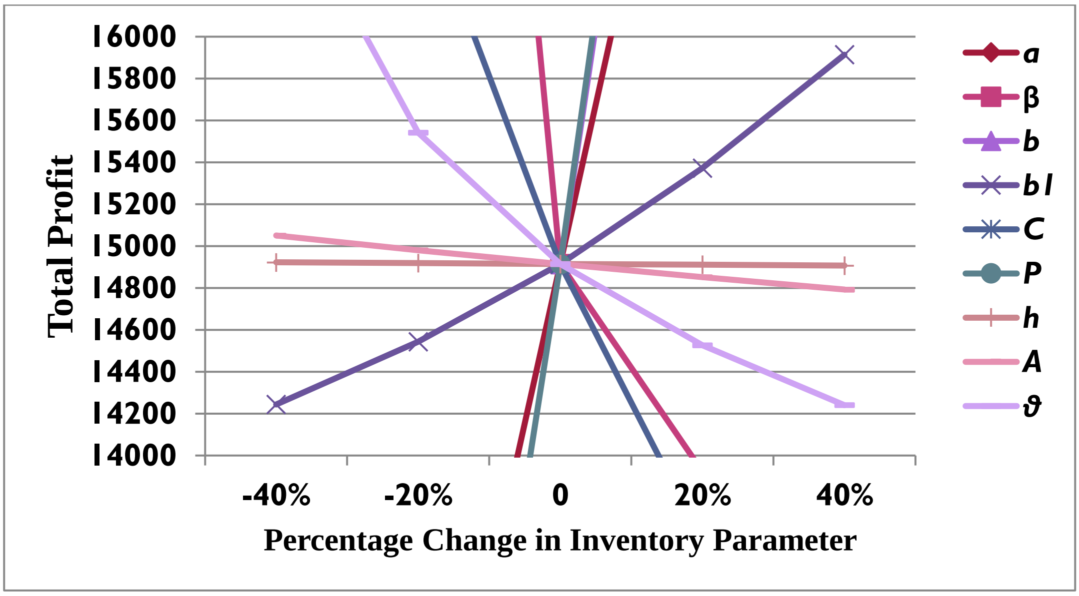

Figure 6 Sensitivity analysis for total profit

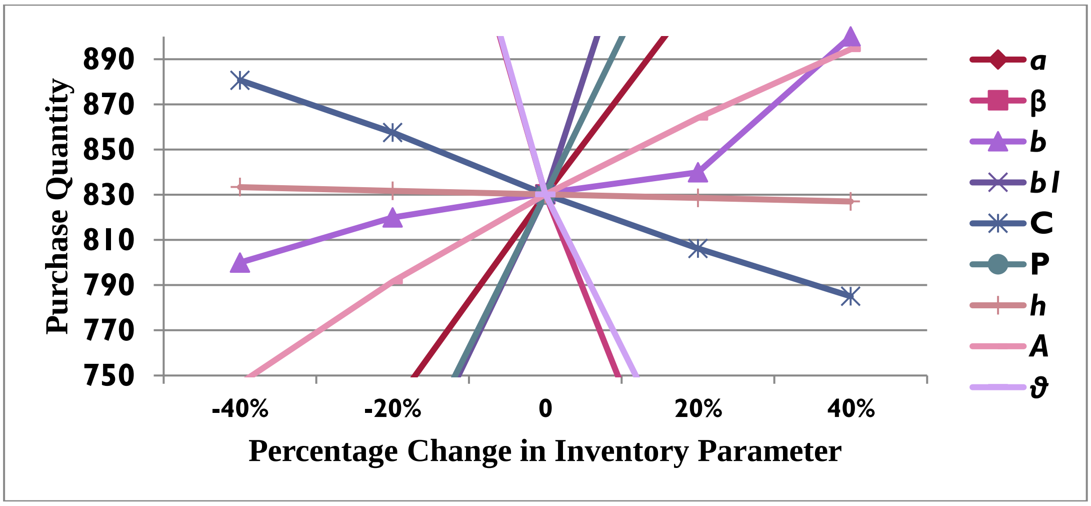

Figure 7 Sensitivity analysis for Purchase quantity (*Q*)

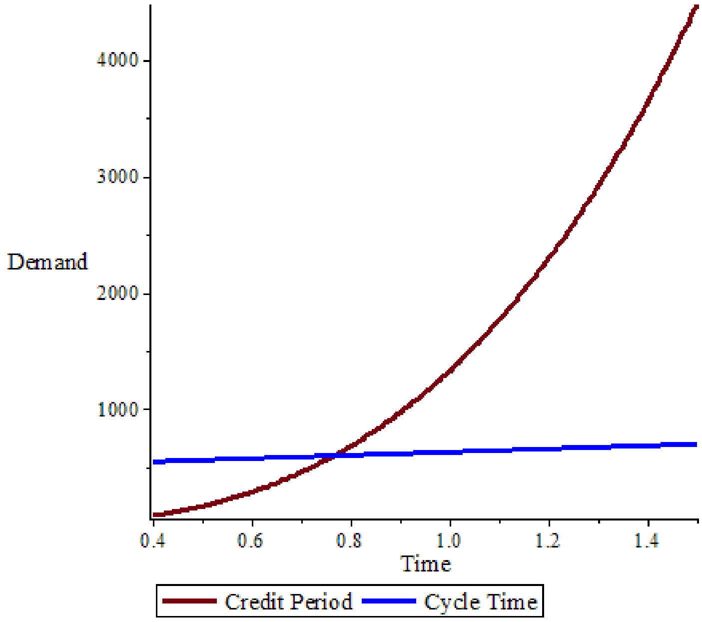

Fig. 8 Demand vs. Time

CONCLUSIONS

In the world of commerce, the choice of setting an appropriate credit period for buyers is an intricate decision, The longer the credit terms, the greater the exposure to the risk of default. The dilemma deepens as demand escalates steadily over time, making the seller's task all the more delicate. In this study, we embark on an exploration of these complexities, situated within the framework of the Economic Order Quantity (EOQ) model, tailored for perishable goods. Here, we derive a non-linear profit function. Necessary conditions are stated to arrive at an optimal solution, guided by empirical data. The findings suggest that as demand scales and selling prices rise, the seller may be encouraged to extend credit periods, yielding larger orders and, ultimately, greater profitability. This research offers valuable insight, providing the seller with a strategic compass for determining the most advantageous credit period.

In future, this concept invites further expansion into stochastic or fuzzy demand, where uncertainty and variability may reshape the seller's approach. Additionally, the potential inclusion of shortages or the integration of innovative preservation techniques to mitigate deterioration opens new avenues for exploration, promising deeper understanding and refined decision-making.

Acknowledgements
The authors are thankful to all the reviewers and editor for their constructive suggestions.

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